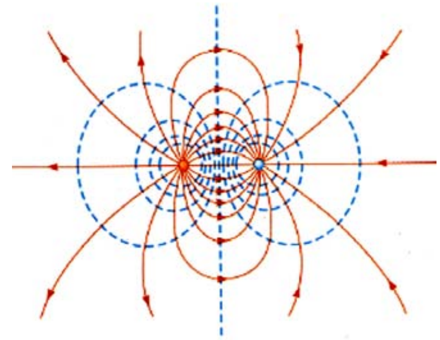
Equipotentials and Electric Fields



Introduction The first topics that we covered in class this semester had to do with electric forces acting on charges and the electric field that caused them. Both electric force, **F**, and *electric field*, **E**, are vectors and they are linked by **F**=q**E** (text eq. [21-3]) where q is the electric charge experiencing the force. It would be nice if we could investigate an electric field caused by a charge distribution by putting a charge on a probe and measuring the force on it. Unfortunately, that is very difficult to do. Fortunately, there is another, albeit indirect, way to determine **E**. As you may already know, **E** is related to the *electric potential*, V. Consequently, **E** can be determined because V can be deduced from voltage measurements since the electric potential is related to the electric potential difference (voltage). In this lab, we will determine the electric field caused by three different charge distributions.

Theory At the bottom of page 602 it is pointed out that *the electric field in a direction is equal to the negative of the derivative of the electric potential with respect to the coordinate in that direction*. For example, the electric field in the direction of an infinitesimal displacement d***l*** is given by

Text eq. [23-8] $$E_l = -\frac{dV}{dl} \qquad (1)$$

Consequently, if there is an electric field in some direction, the electric potential must be changing in that direction. The minus sign implies that *the electric field is in the direction of decreasing electric potential*. Notice that if the electric potential is constant in some direction, there is no electric field in that direction. We refer to the region where the electric potential is not changing (is constant) as an *equipotential*. Since there is no electric field along an equipotential, *the electric field must be perpendicular to an equipotential*.

In this lab we will determine the electric potential from voltage measurements. Let us describe this three ways.

(1) The **general** definition of our approach is as follows. *The electric potential is equal to the electric potential difference (voltage) if a reference potential is used* ***and*** *the reference potential is defined to be zero*. If there is a reference potential, we will always measure the voltage relative to a position that has that potential.

(2) We can make this more explicit by using the first equation from the first lab. Remember that the voltage measured from point **a** to point **b**, V_{ba}, is given by

Voltage (Potential Difference) equals the Potential at **b** if $V_a = 0$

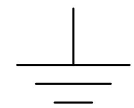
Potential at Point **b** (Red Lead at Point **b**)

Potential at Point **a** (Black Lead at Point **a**)

$$V_{ba} = V_b - V_a$$

It is probably apparent that $V_{ba} = V_b$ if $V_a = 0$. Consequently, we let V_a be the reference potential and assign it a value of zero i.e. $V_a \equiv V_{ref} \equiv 0$. In that case, the electric potential at *any point* **b** is given by the reading on the multimeter and thus we let $V \equiv V_b$.

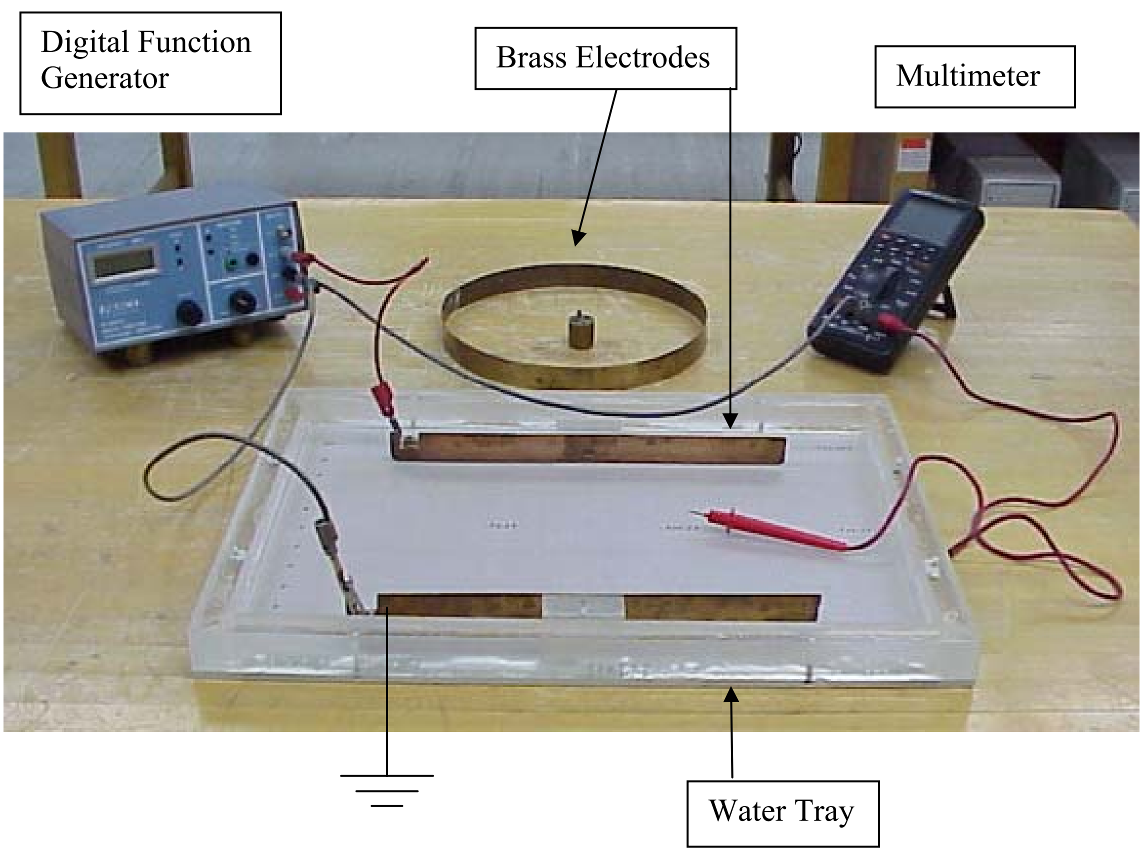
The most frequently used "reference potential" is one that is familiar to us and that is *ground*. This has the special symbol shown at the right. Because it is a conductor on average, the Earth is an equipotential and because of the large size of the Earth its potential remains approximately constant. (Adding or subtracting ordinary amounts of charge to the Earth changes the electric potential of the earth very little.) Furthermore, the electric potential of the Earth is usually defined to be zero. We will follow that convention in this lab.



$V_{ref} \equiv 0$

(3) Finally, we give an **operational** definition of electric potential. When we use a multimeter, if we place the black ("common") lead at the same position for *all* measurements, the red ("test") lead yields a reading of electric potential.

Equipment We will create an electric field by redistributing charges on brass electrodes so that one has a net positive charge while the other has a net negative charge of equal magnitude. Two sets of the brass electrodes (circles and parallel "strips") are shown in the next picture.



The parallel "strips" are shown positioned for measurement and are connected to the digital function generator. We will use the digital function generator (source of *emf*) to cause the redistribution of the charges on the brass electrodes and we will use a multimeter to make voltage measurements in the region between the electrodes.

As was described in the "Theory" section, the black lead will be fixed to one electrode and is the reference. Again, if the black lead is grounded, the multimeter will give values of the electric potential at the position of the red lead. The multimeter requires a small amount of current for its operation and thus it is necessary for the leads to make contact to a conducting material. Consequently, we will put some water between the electrodes. In order to accomplish this, the electrodes are placed in a tray that will hold the water.

Initial Preparations and Further Details of the Experiment

1. Place about ¼ inch of water in the bottom of the water tray. Since the charges only move in the plane of the water tray, the experiment is two-dimensional (plane of the water tray) and thus simulates a cross section of electrodes and electric fields that are infinite perpendicular to the water tray. For example, a circle on the water tray should behave like an infinite cylinder.

2. Connect a long lead (wire) to the terminal on the digital function generator marked GND for "ground." GND is connected the "third prong" on the plug that goes into the power strip (or wall) and that prong is eventually connected to the Earth.

3. Connect another long lead to the terminal marked HI Ω.

In this experiment, we will use the digital function generator to provide a *sinusoidal* (AC) voltage. A sinusoidal voltage is used because water is an ionic conductor. If a constant voltage (DC) were used, the positive ions would pile up at the negative electrode and the negative ions would pile up at the positive electrode because ions cannot enter the brass electrodes. This redistribution of charge (and chemical reactions) would mess up the electric potential and thus we alternate the voltage to prevent painful ion buildup. Fortunately, the digital multimeter can handle alternating voltages.

4. Turn the dial on the multimeter to $\widetilde{\mathrm{V}}(\mathrm{dB})$. On this setting the meter measures a special, average value called the "rms" or root-mean-square value. (Root-mean-square voltage is discussed in Section 25-7 of the textbook.) The meter squares the voltage, averages the square of the voltage then takes the square root of the average of the square. *For the purpose of this experiment, we will just treat this value as a standard constant voltage reading.*

R1: Why does the multimeter not just measure the average value of the voltage? (Hint: What is the average value of a sine wave?)

5. Connect a black lead between the COM input to the multimeter and the GND of the digital function generator. This "grounds" the black lead and thus establishes our reference for the measurements.

6. Connect the red multimeter lead (metal point on one end) to the "μA·V" input of the multimeter. This lead will be used as the probe to measure the electric potential at various points in the water tray. In order to make a measurement, we simply touch the probe onto the point in the water tray at which we wish to measure the electric potential. It is best to hold the

probe vertical when making measurements or the reading won't be accurate. Also, it is important to hold the probe still until the reading is steady. With a little practice it will be easy to trace out the points that have the same potential.

Part 1. The electric field between two parallel strips (plates) The first configuration that we will investigate is two parallel plates, one of which has a net positive charge and the other has a net negative charge of equal magnitude. This configuration gives a relatively simple electric field pattern, and represents a parallel plate capacitor. The goal of Procedure 1 is to determine the magnitude and direction of the electric field in the middle portion the plates. The goal of Procedure 2 is to determine the shape and distribution of the equipotentials and then to use them to deduce the electric field.

Procedure 1

1. Arrange the plates (strips) as shown on **Graph 1** toward the end of this write-up. Be sure that there is enough water in the tray to cover the bottom of the strips. Also, if the strips are badly corroded, it may be useful to use a green abrasive pad to remove some of the corrosion. (The green abrasive pad should be somewhere in the laboratory.)

2. Connect the long lead from the GND input of the digital function generator to the electrode (strip) at $y = 1$.

3. Connect the long lead from the HI Ω input of the digital function generator to the electrode at $y = 8$. It is important to make good connections to the electrodes so you might consider removing some corrosion from the electrodes at the contact points.

4. Turn on the digital function generator. Be sure that the digital function generator is set to provide a sinusoidal voltage. Adjust the frequency to 300 Hz.

5. Turn on the multimeter then touch the probe (red lead from the multimeter) to the electrode connected to HI Ω (the electrode at $y = 8$) and adjust the AMPLITUDE knob on the digital function generator so that the multimeter reads about 5 V. Record the actual value in the space provided.

R2: V_{max} = __________ ± _________ V

6. Measure the electric potential at each of the coordinates listed in the table. R3: Record the values in the table. Remember that the probe should be vertical while a measurement is made.

7. Convert the y-coordinates to meters.
R4: Record the values in the second column.

Coordinate (inches)	y-Coordinate (m)	Electric Potential (V)
(7,2)		
(7,3)		
(7,4)		
(7,5)		
(7,6)		
(7,7)		

8. Plot a graph of electric potential (on the y-axis of the graph) vs. y-coordinate in meters (on the x-axis of the graph). The best way to do that is to use **Excel**. R5: Depending upon the instructions from your instructor, either **Print** the graph or **Save** it as an **Excel** file.

R6: What is the shape of the curve?

R7: What does the shape of the curve tell us about the magnitude of the electric field for the values of y at $x = 7$ (in the "middle" of the plates)?

9. Refer to Eq. (1) and determine the magnitude of the electric field between the parallel plates from the plot. Record the magnitude of the electric field as $|E_{measured}|$ in the space provided.

R8: $|E_{measured}|$ = __________ ± _________ V/m

R9: Explain how you arrived at the value of $|E_{measured}|$.

R10: What is the direction of the electric field? How did you determine the direction of the electric field?

You should have concluded that the electric field in the middle of the plates is uniform (is constant). For this special case, we know that

Text eq. [23-4] $$V_{ba} = -Ed \qquad (3)$$

d is the separation of the electrodes and the V_{ba} is the potential difference between them. For our experiment, $V_{ba} = V_{max}$ and $d = 7$ inches. Use eq. (3) to predict what the magnitude of the electric field should be and record the value as $|E_{predicted}|$) in the space provided.

R11: $|E_{predicted}|$ = __________ ± _________ V/m

R12: Compare $|E_{measured}|$ and $|E_{predicted}|$.

Procedure 2 Our goal for this part of the experiment is to plot (map) equipotentials. Think about what the equipotentials should look like given the shape and symmetry of the electrodes. There are examples of charge distributions and equipotentials on page 600 of the textbook. Proceed as follows.

1. Use the multimeter to measure the electric potential at the middle of the plates i.e. at the coordinate (7,4.5). Record the value as V_1 in the space provided. This value should be half of V_{max}.

R13: V_1 = _________ ± ________ V

R14: On **Graph 1**, place a mark at (7,4.5).

2. Use the multimeter to locate several points to the right of (7,4.5) where the electric potential is the same (V_1). R15: Place marks on **Graph 1** at those points. *Proceed to the right to close to (within about an inch of) the edge of the water tray.*

R16: On **Graph 1**, draw an equipotential by sketching a smooth curve through the dots. (Yes, this equipotential is boring. The next ones are more interesting, at least at the edge of the plates and beyond.)

Next, we will plot equipotentials for two potentials between V=0 and V=V_1. You may wish to use the values V_2 and V_3 as defined by the following arithmetic operations. Fill in the blanks with the appropriate values.

R17: $V_2 = \frac{1}{3}V_1 =$ _________ ± ________ V R18: $V_3 = \frac{2}{3}V_1 =$ _________ ± ________ V

3. Use the multimeter to locate several points with the voltages V_2 and with the voltage V_3. R19: Place marks on **Graph 1** at those points. *Again, proceed to the right to close to (within about an inch of) the edge of the water tray.*

R20: Using symmetry, on **Graph 1**, extrapolate the equipotentials from the middle of the strips to the left.

R21: Using symmetry, sketch equipotentials on **Graph 1** for $V_4 = \frac{4}{3}V_1$ and $V_5 = \frac{5}{3}V_1$.
(Alternatively, you may wish to carry out step 3 for V_4 and V_5.)

R22: Once you have mapped out the equipotentials, on **Graph 1**, sketch in a representation of the electric field. Your instructor will tell you whether to draw electric field vectors (individual vectors at each point) or electric field lines. Remember that the electric field must be perpendicular to the equipotentials and is directed from high potential to have low potential.

R23: On **Graph 1**, sketch the charge distribution on each plate i.e. draw charges on the plates.

Part 2. The electric field between concentric cylinders In this part of the lab we will investigate a slightly less simple configuration, a small, solid cylinder and a large circular ring. This should simulate a cross section of a long wire and coaxial cylinder.

Procedure 1

1. Replace the plates with the small solid brass cylinder and the large circular ring in the water tray so that they are both centered at (7,5). The placement of the objects is shown on **Graph 2** at the end of this write-up. If the large cylinder looks badly out of round, you may need to reshape it a little.

2. Connect the GND input on the digital function generator to the small, central electrode. (Be sure that the COM input of the multimeter continues to be connected to the GND input on the digital function generator.)

3. Connect the large outer electrode to the gray HI Ω output on the digital function generator.

4. Touch the probe (red lead of the multimeter) to the large ring and adjust the digital function generator so that the multimeter reads about 5 V. Record the actual value as V_{max} in the space provided.

R24: V_{max} = __________ ± _________ V

5. Determine the potentials for four equipotentials by doing the following arithmetic. Fill in the blanks with the values of the equipotentials.

R25: $V_1 = \frac{1}{5} V_{max} =$__________ ± _________ V R26: $V_2 = \frac{2}{5} V_{max} =$__________ ± _________ V

R27: $V_3 = \frac{3}{5} V_{max} =$__________ ± _________ V R28: $V_4 = \frac{4}{5} V_{max} =$__________ ± _________ V

R29: On **Graph 2**, map out the equipotentials for these potentials (V_1, V_2, V_3 and V_4). It should not take many measurements for each potential before you are able to sketch the equipotential.

R30: On **Graph 2**, sketch the electric field between the electrodes and the charge distribution on the electrodes.

R31: Predict what the electric field should be outside both electrodes.

R32: Describe an experiment that could be carried out to verify your prediction in R31.

6. Carry out the experiment that was described in R32.

R33: Discuss the results of this experiment.

7. Place **Graph 1** and **Graph 2** side by side then read the following two sentences carefully. Remember that in both cases the equipotentials are *equally spaced in **potential***. However, note that while the equipotentials are also *equally spaced in **space** in the middle of the plates*, they are *not equally spaced in **space** anywhere for the coaxial cylinders*.

R34: Explain what the previous two sentences imply concerning the magnitude of the electric field between the cylinders.

Theory Next, we will take data that should allow us to determine how the electric potential depends on r, the distance from the center of the arrangement. The theory is as follows. You probably know that the electric field outside a long uniform line of charge with charge per length, λ, is (Examples 22-5 and 21-10 in the textbook)

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r} \qquad (4)$$

Next, following the procedures introduced in Section 23-2 of the textbook, we will calculate the electric potential for our configuration. We start by using the equation for potential difference

$$V_\mathrm{b} - V_\mathrm{a} = -\int_\mathrm{a}^\mathrm{b} \mathbf{E} \bullet d\boldsymbol{l} = -\int_\mathrm{a}^\mathrm{b} E dr \qquad (5)$$

For us, $V_\mathrm{a} = 0$, a = the radius of the small cylinder and $V_\mathrm{b} = V$ is the potential at any radius r. Consequently, our equation for the electric potential becomes

$$V = -\frac{\lambda}{2\pi\epsilon_0}\ln(r) + \frac{\lambda}{2\pi\epsilon_0}\ln(\mathrm{a}) \qquad (6)$$

Thus, the electric potential should vary as the $\ln(r)$. We will now test this theory.

Procedure 2

1. Measure the electric potential, V, at the coordinates listed in the first column of the next table. R35: Record the measured values of V in the last column of the table.

2. Carefully transform the coordinates to the distance from the *center* of the center cylinder. R36: Record the values in the second column of the table as the radius, r. Change the units of r from inches to meters. R37: Record those values in the third column of the table.

3. Calculate the natural logarithm of the radius.

R38: Record the values of $\ln(r)$ in the fourth column of the table.

Coordinate (inches)	Radius, r (inches)	Radius, r (m)	$\ln(r)$	Potential, V (V)
(7,1)				
(7,1.5)				
(7,2)				
(7,2.5)				
(7,3)				
(7,3.5)				
(7,4)				

4. Plot the potential, V, (on the y-axis of the graph) vs. $\ln(r)$ (on the x-axis of the graph). If eq. (6) is valid, the plot should be a straight line.

R39: Depending upon the instructions from your instructor, either **Print** the graph or move it so that it is to the right of the previous graph in your **Excel** file.

5. Obtain a best-fit straight line to this graph. Use the best-fit parameters to calculate the effective linear charge density (charge per length) on the center cylinder. (Hint: According to Eq. (6), the slope of the V vs $\ln(r)$ graph should equal $-\frac{\lambda}{2\pi\epsilon_0}$.) Record the values in the space provided.

R40: Slope of graph = ___________________ R41: λ = ______________ C/m.

R42: Is the sign of λ what you expect? Explain.

Part 3. The electric field between a hollow cylinder and a plate The purpose of this section is to investigate the equipotentials and the electric field inside and outside a hollow cylindrical conductor near an infinite plate.

Procedure

1. Replace the coaxial cylinders with the medium cylinder and one plate as shown on **Graph 3**.

2. Connect the cylinder to the HI Ω terminal on the digital function generator.

3. Connect the plate to the GND input on the digital function generator. (Be sure that the COM input of the multimeter continues to be connected to the GND input on the digital function generator.)

4. Touch the probe to the cylinder and adjust the digital function generator to about 5 V. Record the value in the space provided.

R43: V_{max} = __________ ± _________ V

5. Measure the electric potential at several points spread evenly throughout the space inside the hollow cylinder electrode.

R44: Comment on the variation of the potential readings at these points. What is the electric field inside the hollow cylinder?

4. Choose four potentials that are approximately evenly spaced between zero and V_{max}.

R45: Plot equipotentials for those potentials on **Graph 3**. Label each equipotential with the value of the potential. Be sure that you take enough data that the shape of the equipotentials is clear.

R46: On the plot, sketch the electric field and the charge distribution on each electrode.

Suggestions for further work:

1. Measure the electric potential vs. *y*-coordinate at the edge ($x = 15$) of the parallel plate electrodes. R47: Record the results in the table, make a plot and compare the results with that for the electric potential vs. *y*-coordinate at the middle of the plates.
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Coordinate (inches)	*y*-Coordinate (m)	Electric Potential (V)
(15,2)		
(15,3)		
(15,4)		
(15,5)		
(15,6)		
(15,7)		

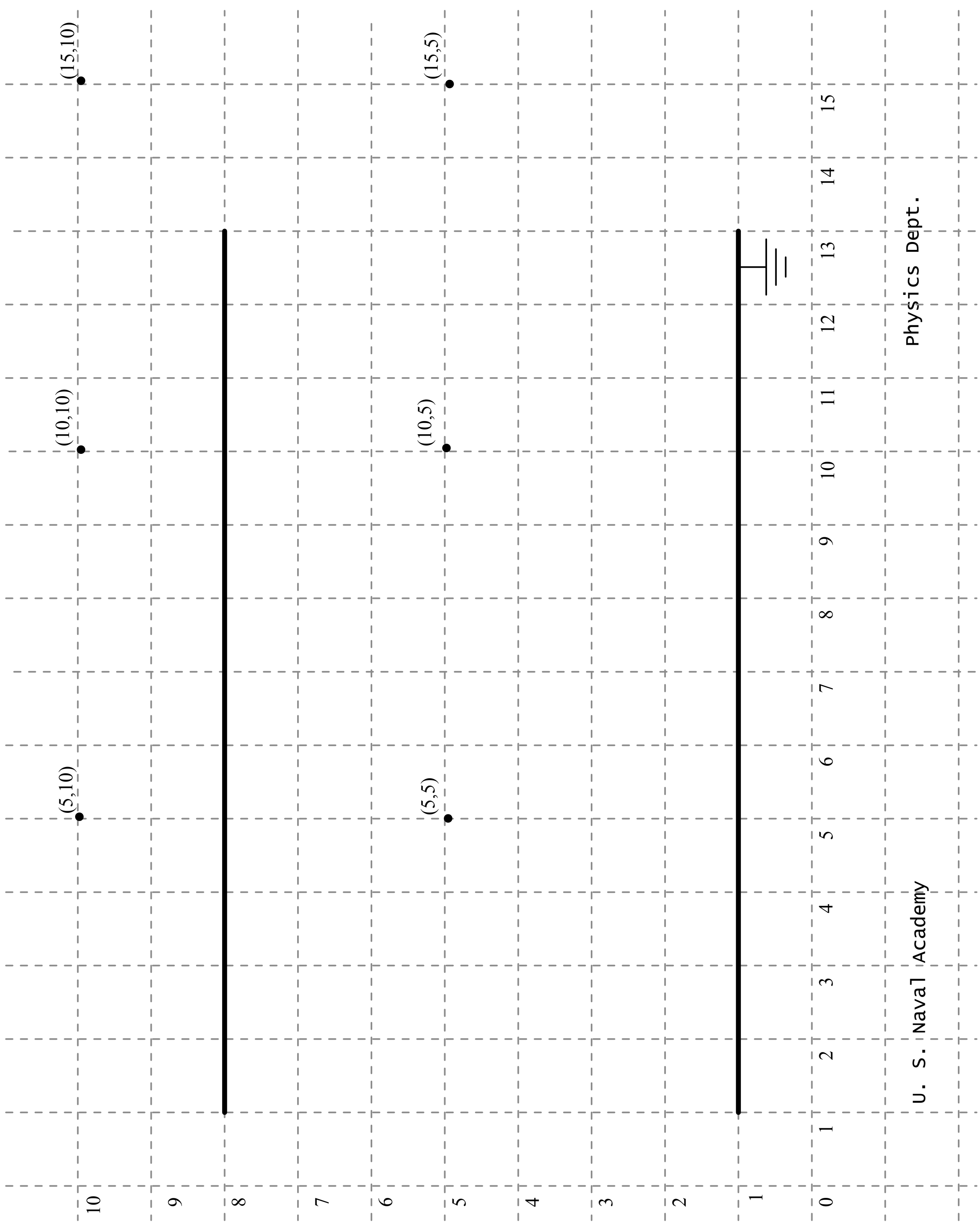
2. Choose other configurations using either the straight or the cylindrical electrodes. R48: Map out several equipotential lines between the electrodes and sketch the electric field.

End of Lab Checkout

1. Empty the water tray.

2. Disconnect any circuits and tidy up the workstation.

Graph 1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0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
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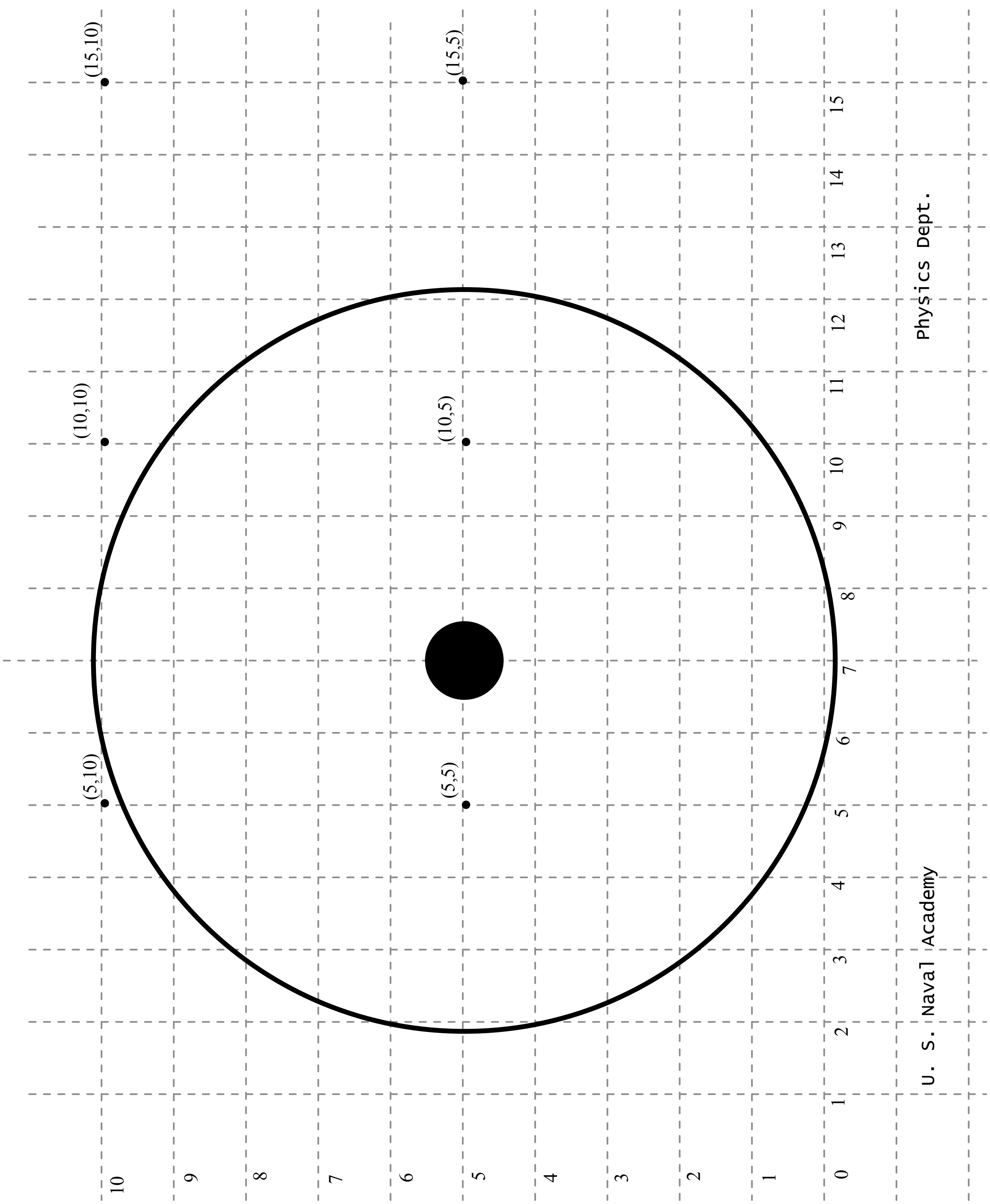
Graph 2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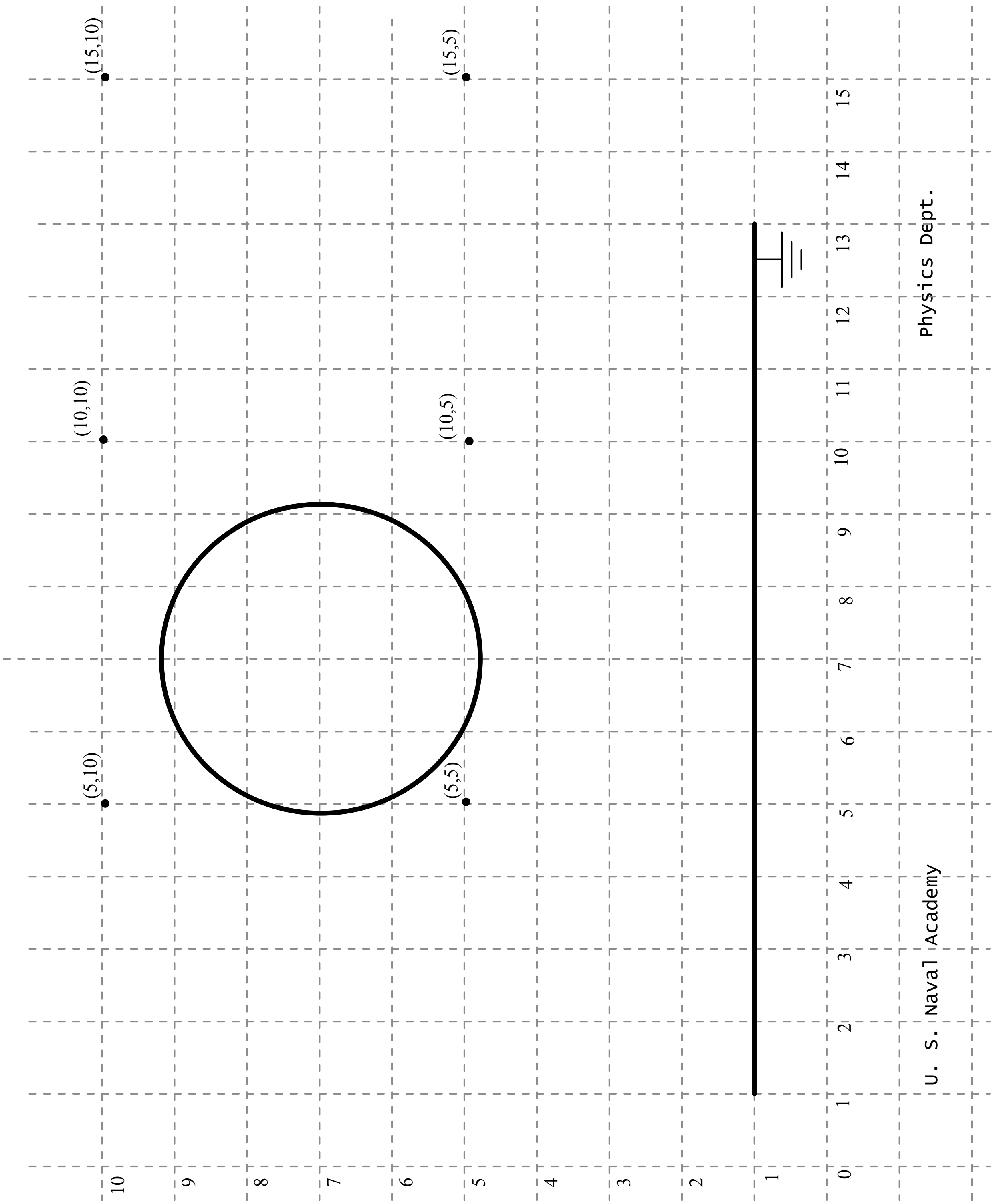
Graph 3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